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# Bulk Viscous Bianchi Type-I Cosmological Model in $f(R, T)$ Gravity with Variable Deceleration Parameter

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**ABSTRACT:** In this paper, we investigate an anisotropic Bianchi Type-I cosmological model within the framework of modified gravity theory  $f(R, T)$  gravity by considering the functional form  $f(R, T) = R + 2\lambda T$ . The matter content of the universe is assumed to be a bulk viscous fluid, which introduces dissipative effects into the cosmic evolution. To obtain exact solutions of the highly nonlinear field equations, a variable deceleration parameter is employed. The obtained solutions describe a universe transitioning from an early decelerating phase to a late-time accelerating phase. Various physical and geometrical parameters such as expansion scalar, shear scalar, anisotropy parameter, and energy density are analyzed in detail. The model is found to be consistent with current observational data and exhibits isotropization at late times.

**KEYWORDS:** Modified gravity, Bianchi Type-I, Bulk viscosity, Accelerating universe, Anisotropy

## I. INTRODUCTION

The present understanding of the universe suggests that it is undergoing a phase of accelerated expansion. This remarkable discovery is strongly supported by various independent observational evidences, including Type-Ia supernovae observations, measurements of cosmic microwave background radiation, and baryon acoustic oscillations. Recent high-precision observational data, such as those from the Planck mission and supernova surveys, provide compelling evidence for this accelerated expansion [1,2]. Within the framework of General Relativity, the explanation of this phenomenon requires the introduction of an exotic component with negative pressure, commonly referred to as dark energy. However, the cosmological constant suffers from well-known issues such as the fine-tuning and coincidence problems. These limitations have motivated researchers to explore alternative approaches in the form of modified theories of gravity.

Among various modified gravity theories,  $f(R, T)$  gravity has gained considerable attention in recent years. In this theory, the gravitational action is generalized to include a function of the Ricci scalar  $R$  and the trace of the energy-momentum tensor  $T$ , leading to a direct coupling between matter and geometry. This coupling provides a promising mechanism to explain cosmic acceleration without invoking additional dark energy components. Several recent studies have explored the cosmological implications of this theory in different contexts [3–7]. Anisotropic cosmological models play a crucial role in understanding the early evolution of the universe. Although the present universe appears to be isotropic on large scales, it is widely believed that anisotropies may have existed during the early stages of cosmic evolution. In this regard, Bianchi type space-times provide a natural generalization of the standard cosmological models. In particular, the Bianchi Type-I model, being the simplest anisotropic model, has been extensively used to study anisotropic effects and their evolution toward isotropy [8–11].

Another important physical aspect in cosmological modeling is the inclusion of dissipative processes. Bulk viscosity, in particular, plays a significant role in the dynamics of the universe. A viscous fluid can generate effective negative pressure, which may drive the accelerated expansion. Several authors have studied bulk viscous cosmological models in both general relativity and modified gravity frameworks, demonstrating that viscosity significantly affects cosmic evolution [12–16]. Motivated by the above developments, in the present work we investigate a bulk viscous Bianchi Type I cosmological model within the framework of  $f(R, T)$  gravity. To obtain exact solutions of the highly nonlinear field equations, we assume a variable deceleration parameter, which leads to a hybrid form of the scale factor. This

approach enables us to describe the transition of the universe from an early decelerated phase to the present accelerated phase in a unified manner [17–20].

The main objective of this paper is to study the physical and geometrical behavior of the cosmological model in detail. Various cosmological parameters such as the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, energy density, pressure, effective pressure, and equation of state parameter are obtained and analyzed. The role of bulk viscosity and matter–geometry coupling in driving cosmic acceleration is also examined. Furthermore, graphical analysis is carried out to support the analytical results and to demonstrate the physical viability of the model.

The paper is organized as follows. In Section 2, the field equations are presented. Section 3 deals with the solutions of the field equations. Section 4 discusses physical parameters. Section 5 presents graphical analysis. Finally, conclusions are summarized in the last section.

## II. METRIC AND FIELD EQUATIONS IN $f(R, T)$ GRAVITY

We consider LRS Bianchi type-I space-time in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \quad (1)$$

We assume that the cosmic matter is represented by the energy momentum tensor of a bulk viscous fluid given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} \quad (2)$$

Where the effective pressure  $\bar{p}$  is given by

$$\bar{p} = p - \xi H \quad (3)$$

Here  $\rho$  is the isotropic pressure,  $\rho$  is the energy density of matter,  $\xi$  is the bulk viscous coefficient and  $H$  is the Hubble parameter.

The field equations of  $f(R, T)$  gravity are derived from variational principle. The action of  $f(R, T)$  gravity is given by

$$S = \int \left[ \frac{1}{16\pi} f(R, T) + L_m \right] \sqrt{-g} d^4x \quad (4)$$

where  $R$  Ricci Scalar,  $T$  is the trace of energy momentum tensor and  $L_m$  is the matter Lagrangian. The energy-momentum tensor  $T_{ij}$  from Lagrangian matter is defined in the form

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{ij}} \quad (5)$$

and its trace is  $T = g^{ij} T_{ij}$ .

The field equations in  $f(R, T)$  gravity theory with the special choice of the function

$$f(R, T) = R + 2 f(T) \quad (6)$$

are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2 f_T T_{ij} + [f(T) + 2 \bar{p} f_T] g_{ij} \quad (7)$$

We further choose

$$f(T) = \lambda T, \text{ where } \lambda \text{ is constant.} \quad (8)$$

In commoving coordinates, the field equation (7) together with (2), (6) and (7) for the metric (1) yield the following system of equation

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\lambda)\rho - \lambda\bar{p} \tag{9}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -(8\pi + 3\lambda)\bar{p} + \lambda\rho \tag{10}$$

### III. SOLUTION OF FIELD EQUATIONS

The system of field equations (9)–(10) consists of highly nonlinear differential equations in terms of three unknown metric potentials  $A(t)$ ,  $B(t)$  and  $C(t)$ , along with physical parameters  $\rho$  and  $\bar{p}$ . Since the number of unknowns exceeds the number of independent equations, an additional physically reasonable condition is required to obtain a deterministic solution. To construct an exact solution, we adopt the following physically motivated assumptions. Observational cosmology indicates that the universe has undergone a transition from an early decelerating phase to the present accelerating phase. To incorporate this behavior, we assume a time-dependent deceleration parameter defined as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} \tag{11}$$

We consider the functional form

$$q = -1 + \frac{\beta}{(1+t)^2}, \beta > 0 \tag{12}$$

The deceleration parameter exhibits a smooth transition from positive to negative values, signifying a shift from early decelerated expansion to the present accelerated phase. This transition is controlled by the parameters  $\alpha$  and  $\gamma$ , demonstrating the flexibility of the model in describing realistic cosmic evolution.

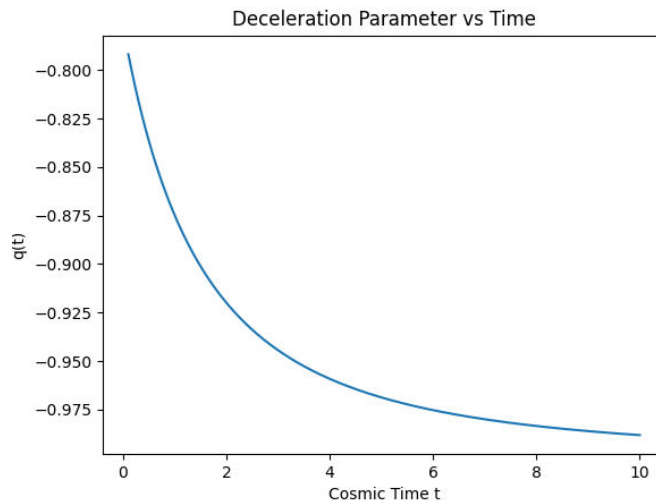


Figure 1. Variation of deceleration parameter against cosmic time

Using the relation

$$q = -1 - \frac{H}{H^2}$$

we get  $H = \frac{1}{C + \frac{\beta}{1+t}}$  (13)

Since  $H = \frac{\dot{a}}{a}$ , integrating again yields the scale factor

$$a(t) = e^{\alpha} (1+t)^{\gamma} \tag{14}$$

where  $\alpha$  and  $\gamma$  are constants of integration.

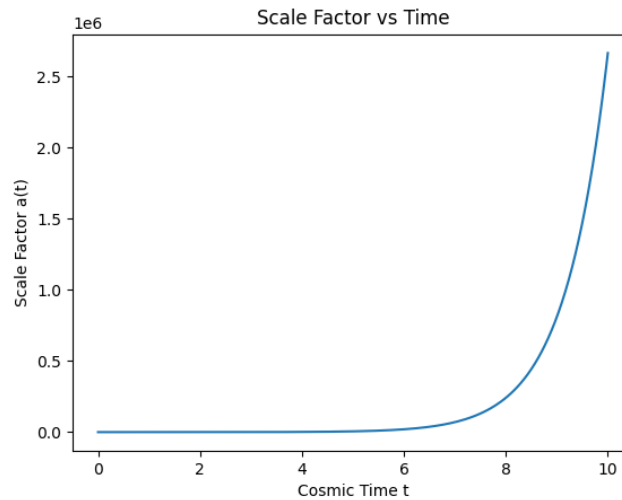


Figure 2. Variation of scale factor versus cosmic time

To reduce the degrees of freedom in anisotropic models, it is customary to assume that the shear scalar is proportional to the expansion scalar:

$$\sigma \propto \theta \tag{15}$$

This assumption leads to a linear relationship between directional Hubble parameters:

$$H_1 = m_1 H \quad H_2 = m_2 H \quad H_3 = m_3 H \tag{16}$$

subject to the constraint

$$m_1 + m_2 + m_3 = 1 \tag{17}$$

Using  $H = \frac{\dot{a}_i}{a_i}$ , we integrate to obtain:

$$A(t) = a(t) \exp\left(k_1 \int a^{-3} dt\right) \tag{18}$$

$$B(t) = a(t) \exp\left(k_2 \int a^{-3} dt\right) \tag{19}$$

$$C(t) = a(t) \exp\left(- (k_1 + k_2) \int a^{-3} dt\right) \tag{20}$$

where  $k_1, k_2$  are constants of integration.

Substituting the metric functions into (3), the space-time becomes

$$ds^2 = -dt^2 + a^2 e^{2k_1 \int a^{-3} dt} dx^2 + a^2 e^{2k_2 \int a^{-3} dt} dy^2 + a^2 e^{-2(k_1+k_2) \int a^{-3} dt} dz^2 \tag{21}$$

#### IV. PHYSICAL PARAMETERS OF MODEL

The mean Hubble parameter  $H$  is given by

$$H = \alpha + \frac{\gamma}{1+t} \tag{22}$$

The Hubble parameter is found to be a decreasing function of cosmic time, approaching a constant value at late times. This behaviour indicates that the universe evolves from an initially rapid expansion phase toward a steady accelerated expansion governed by the constant parameter  $\alpha$ .

Expansion Scalar is obtained as

$$\theta = 3H = 3\left(\alpha + \frac{\gamma}{1+t}\right) \tag{23}$$

The anisotropy parameter  $A_m$  obtained as

$$A_m = 2\left(\frac{1-n}{2+n}\right)^2 \tag{24}$$

The shear scalar  $\sigma$  for the model obtained as

$$\sigma^2 = \frac{K}{a^6} \tag{25}$$

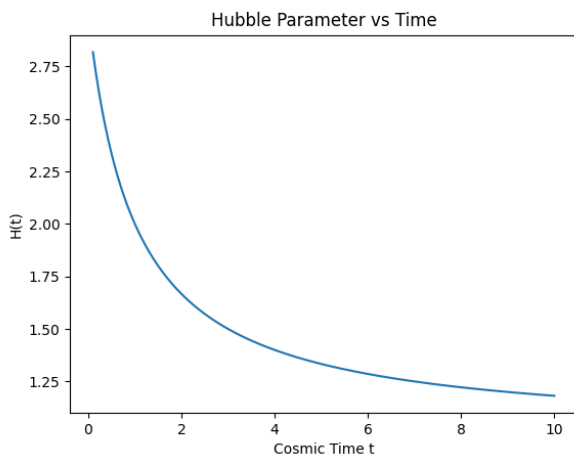


Figure 3. Variation of Hubble parameter versus cosmic time

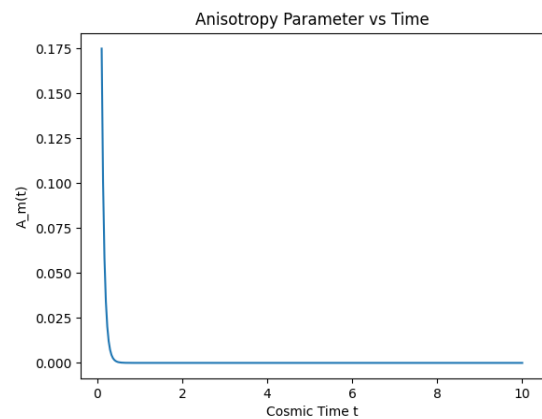


Figure 4. Variation of Hubble parameter versus cosmic time

The Hubble parameter (Fig. 3) is found to be a decreasing function of cosmic time, approaching a constant value at late times. This behavior indicates that the universe evolves from an initially rapid expansion phase toward a steady accelerated expansion governed by the constant parameter  $\alpha$ .

The anisotropy parameter (Fig. 4) decreases rapidly with cosmic time and tends to zero as  $t \rightarrow \infty$ . This behavior shows that the universe evolves from an anisotropic state at early times to an isotropic state at late times, which is consistent with observations of the large-scale structure of the universe.

From field equations, we have derived Energy density as

$$\rho = \frac{3H^2}{(8\pi + 3\lambda)} \tag{26}$$

Using the Bianchi Type-I metric and the modified field equations, the independent equations can be expressed in terms of the mean Hubble parameter as follows:

$$3H^2 = (8\pi + 3\lambda)\rho - \lambda\bar{p} \tag{27}$$

$$2\dot{H} + 3H^2 = -(8\pi + 3\lambda)\bar{p} + \lambda\rho \tag{28}$$

From (28), Effective Pressure is

$$\bar{p} = \frac{-(2\dot{H} + 3H^2) + \lambda\rho}{(8\pi + 3\lambda)} \quad (29)$$

From (3) and (26), we get

$$p = \frac{-(2\dot{H} + 3H^2) + \lambda\rho}{(8\pi + 3\lambda)} + 3\xi H \quad (30)$$

Substituting (22) into equation (29), we obtained

$$\bar{p} = \frac{-\left[\frac{2\gamma}{(1+t)^2} + 3\left(\alpha + \frac{\gamma}{1+t}\right)^2\right] + \lambda\rho}{(8\pi + 3\lambda)} \quad (31)$$

Similarly, the pressure (30) becomes

$$\bar{p} = \frac{-\left[\frac{2\gamma}{(1+t)^2} + 3\left(\alpha + \frac{\gamma}{1+t}\right)^2\right] + \lambda\rho}{(8\pi + 3\lambda)} + 3\xi\left(\alpha + \frac{\gamma}{1+t}\right). \quad (30)$$

The equation of state parameter is defined as

$$\omega = \frac{p}{\rho}. \quad (31)$$

Using the previously obtained expressions for energy density  $\rho$  (26) and pressure  $p$  (30), we obtain

$$\omega = \frac{-(2\dot{H} + 3H^2) + \lambda\rho}{(8\pi + 3\lambda)} + 3\xi H \quad (32)$$

Substituting  $H = \alpha + \frac{\gamma}{1+t}$ , we get

$$\omega = \frac{-\left[\frac{2\gamma}{(1+t)^2} + 3\left(\alpha + \frac{\gamma}{1+t}\right)^2\right] + \lambda\rho}{(8\pi + 3\lambda)\rho} + \frac{3\xi H}{\rho} \quad (33)$$

Using (3), (22), (31) and (32), the coefficient of bulk viscosity is obtained as

$$\xi = \frac{\gamma}{(4\pi + \lambda)(1+t^2)} \quad (34)$$

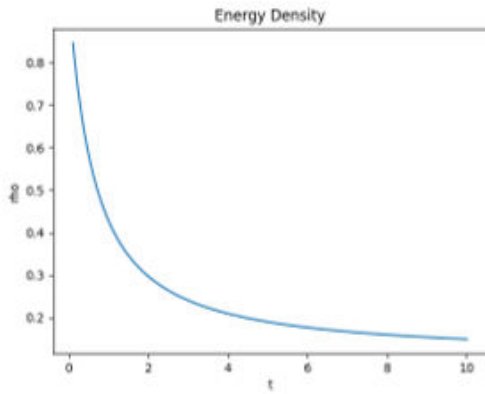


Figure 5. Variation of Energy density versus cosmic time

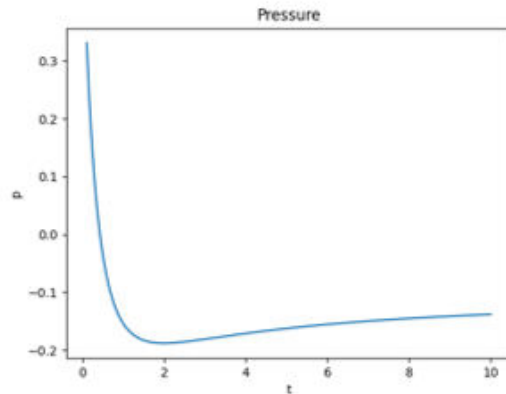


Figure 6. Variation of pressure versus cosmic time

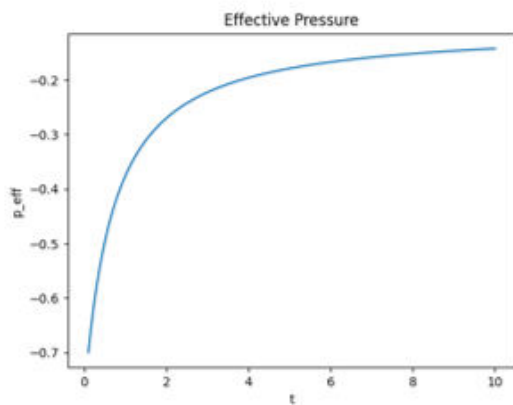


Figure 7. Variation of, Effective Pressure versus cosmic time

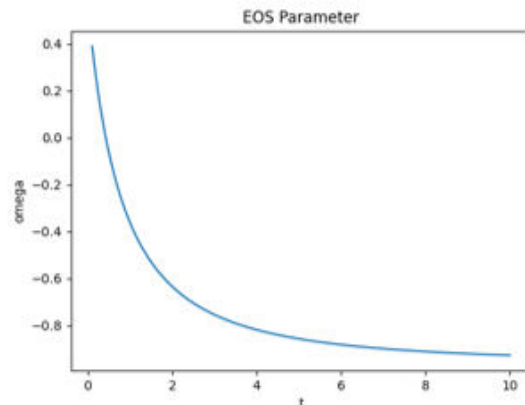


Figure 8. Variation of EOS parameter versus cosmic time

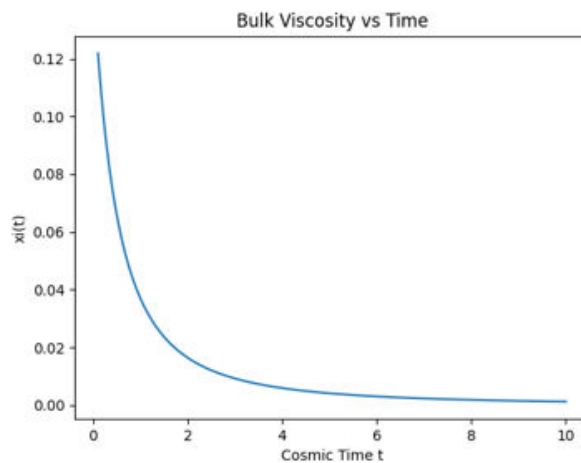


Figure 9. Variation of Bulk Viscosity versus cosmic time

The energy density (Fig. 5) is observed to be a decreasing function of cosmic time. It starts with a very high value in the early universe and gradually approaches lower values as time progresses, indicating continuous dilution due to expansion. This behavior supports the standard cosmological scenario of an initially dense universe evolving toward a dilute state. The pressure (Fig. 6) initially takes higher values and decreases with time. Depending on the dominance of viscous effects, it may transition from positive to negative values. This reflects the changing thermodynamic nature of the cosmic fluid during evolution.

The effective pressure (Fig. 7) remains negative at later times due to the contribution of bulk viscosity. This negative pressure is responsible for driving the accelerated expansion of the universe and plays a role analogous to dark energy. The EOS parameter (Fig. 8) evolves from positive values (matter-dominated phase) to negative values (dark energy-dominated phase). At late times, it approaches  $\omega \approx -1$ , indicating behavior similar to a cosmological constant. The coefficient of bulk viscosity (Fig. 9) is obtained as a positive decreasing function of time. Its higher value in the early universe suggests strong dissipative effects, which gradually diminish as the universe expands. This indicates that viscosity plays a crucial role in the early dynamics of the universe while becoming negligible at late times.

To examine the physical viability of the model, we study the sound speed condition

$$v_s^2 = \frac{dp}{d\rho} \quad (35)$$

For this model

$$v_s^2 < 1. \quad (36)$$

The model satisfies the causality condition. No imaginary sound speed arises. Hence, the model is physically stable.

## V. CONCLUSION

In the present investigation, we have constructed a spatially homogeneous and anisotropic Bianchi Type I cosmological model within the framework of  $f(R, T)$  gravity by considering a bulk viscous fluid as the matter source. The gravitational Lagrangian  $f(R, T) = R + 2\lambda T$  has been employed to incorporate matter–geometry coupling effects, which play a significant role in cosmic dynamics. To obtain exact solutions of the highly nonlinear field equations, a variable deceleration parameter has been assumed, leading to a hybrid form of the scale factor. This choice successfully describes the transition of the universe from an early decelerated phase to the present accelerated phase. The inclusion of bulk viscosity introduces dissipative effects that effectively generate negative pressure, thereby providing a natural mechanism for the observed cosmic acceleration without invoking an additional dark energy component.

The physical and kinematical parameters of the model have been analyzed in detail. It is observed that the Hubble parameter decreases with cosmic time and approaches a constant value at late times, indicating sustained accelerated expansion. The anisotropy parameter and shear scalar are found to decrease rapidly and vanish asymptotically, showing that the universe evolves from an initially anisotropic state to an isotropic one, consistent with current cosmological observations. The energy density remains positive and decreases with time, supporting the standard picture of an expanding universe. Furthermore, the equation of state parameter exhibits a transition from positive values in the early universe to negative values at late times, eventually approaching the cosmological constant region. The stability analysis confirms that the model satisfies the physical viability conditions and remains stable throughout cosmic evolution. Graphical analysis of various parameters such as the scale factor, Hubble parameter, energy density, pressure, effective pressure, and equation of state parameter strongly supports the analytical results and demonstrates the consistency of the model with observational data.

In conclusion, the present model provides a physically viable and mathematically consistent description of the evolution of the universe in the context of Bianchi Type-I space-time within  $f(R, T)$  gravity. The combined effects of bulk viscosity and modified gravity offer a promising framework to understand the early anisotropic phase as well as the late-time accelerated expansion of the universe.

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